A beautiful proof of a probably useless theorem.

by

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Let a ~~non-negative~~ positive continuous function be defined on the Euclidean plane, such that its integral taken over the whole plane is finite, say = 1. Then there exists a set of rectangular axes such that the four integrals of the function, taken over the four quadrants respectively, are all equal to each other.

For a fixed orientation of the cross we can first move it in a direction perpendicular to one axis, such that eventually the surface integrals taken over the half planes at both sides of that one axis are equal to eachother, a state of affairs that is not destroyed by moving thereafter the cross in a direction perpendicular to the other axis until also for that other axis the surface integrals at both sides are equal to eachother. Such a so-called "balanced position" always exists and is uniquely determined by the orientation of the cross because the function integrated is ~~non-negative~~ positive. In a balanced position the surface integrals taken over opposite quadrants are equal: if they are both $\frac{1}{4}$ + alpha , then the other two must each be $\frac{1}{4}$ - alpha .

While keeping the relation "the cross is in a balanced position" invariant, the cross is now slowly rotated.The continuously changing value of the surface integral, taken over the quadrant encompassed by two chosen half-axes, that was originally $= \frac{1}{4}$ + alpha , must have been at least once $= \frac{1}{4}$ before we have rotated the cross over more than a right angle, for after a rotation over a right angle the cross has returned to its original balanced position, in which the continuously observed surface integral has the value $\frac{1}{4}$ - alpha .

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