

Potter's proof of disjunction's symmetry

Dr. Walter M. Potter of Southwestern University, Georgetown, TX 78626, taught me that the symmetry of the disjunction can be derived from my other axioms:

$$\begin{aligned}
 & P \equiv Q \\
 = & \quad \{ \text{idempotence of } \vee \} \\
 & (P \equiv Q) \vee (P \equiv Q) \\
 = & \quad \{ \vee \text{ distributes forwards over } \equiv \} \\
 & (P \equiv Q) \vee P \equiv (P \equiv Q) \vee Q \\
 = & \quad \{ \vee \text{ distributes backwards over } \equiv, \text{ twice} \} \\
 & P \vee P \equiv Q \vee P \equiv P \vee Q \equiv Q \vee Q \\
 = & \quad \{ \text{idempotence of } \vee, \text{ twice} \} \\
 & P \equiv Q \vee P \equiv P \vee Q \equiv Q \quad ;
 \end{aligned}$$

From the first and the last line (and the properties of \equiv) follows

$$[Q \vee P \equiv P \vee Q]$$

Q.E.D.

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